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Noise Analysis of Microwave Circuits with General Topology

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ABSTRACT

A method for noise analysis of microwave multiports with general internal topology is presented. The multiport circuit is seperated into the connection circuit and the circuit elements. Based upon the digraph representation of the connection circuit the scattering matrix of the connection circuit is computed from the fundamental cut set matrix and the fundamental loop matrix of the connection circuit. From this and the S–parameters of the circuit elements and the correlation spectra of the internal noise sources the S–matrix of the multiport and the correlation matrix of its external equivalent noise sources may be determined directly. Circuit elements may be noisy two–terminal elements as well as noisy n–terminal elements and noisy multiports.

1 Introduction

A new method for the topological analysis of linear noisy multiports with arbitrary internal topology is presented. A method for evaluating the scattering parameters of linear n–ports with general internal topology based on a seperation of the n–port circuit into the connection circuit and the circuit elements has been described in [1]. The noise analysis is based on the correlation matrix description of the noise power spectra [2,3]

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2 The Topological Analysis of the Circuit

We consider linear noisy multiports with arbitrary internal lumped circuit structure. The circuit elements are two–terminal elements, n–ports and grounded n–terminal elements. The circuit may be seperated into a connection circuit and the circuit elements. The connection circuit contains a branch for each external port of the multiport and for each port of a circuit element. The connection circuit is represented by a digraph (directed graph). The branch currents and branch voltages of the connection circuit are represented by the vectors $\mathbf{V}$ and $\mathbf{I}$ respectively. The topological analysis of the connection circuit is based on the cut–set analysis and the loop analysis.

As an example we consider the amplifier circuit with two amplifier stages paralleled via Wilkinson power dividers shown in Fig. 1. The corresponding circuit graph is shown in Fig. 2. The circuit graph exhibits three components with the datum nodes d_1, d_2 and d_3. The direction of the branches is choosen arbitrarily and the directions of the loops and cut sets is determined by the assocciated links and twigs respectively.

Following [1] we introduce the *extended fundamental loop matrix* $\mathbf{K}_E$ and the *extended fundamental cut set matrix* $\mathbf{Q}_E$.

$$(\mathbf{K}_E)_{\nu\mu} = \begin{cases} 1 & \text{if branch } \mu \text{ is a link in loop } \nu \text{ and has the same reference direction as loop } \nu \\ -1 & \text{if branch } \mu \text{ is a link in loop } \nu \text{ and has opposite reference direction as loop } \nu \\ 0 & \text{if branch } \mu \text{ is no link or not in loop } \nu \end{cases} \quad (1)$$

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$$(\mathbf{Q}_E)_{\nu\mu} = \qquad (2)$$

$$\begin{cases} 1 & \text{if branch } \mu \text{ is a twig belonging to cut set } \nu \\ & \text{and has the same direction as cut set } \nu \\ -1 & \text{if branch } \mu \text{ is a twig belonging to cut set } \nu \\ & \text{and has the opposite direction as cut set } \nu \\ 0 & \text{if branch } \mu \text{ is not in cut set } \nu. \end{cases}$$

For a circuit graph with b branches the dimension of $\mathbf{K}_E$ and $\mathbf{Q}_E$ is $b \times b$. If the circuit graph has n nodes and consists of k components any forest of the circuit graph exhibits $n-k$ twigs and $l = b-n+k$ link. The *Kirchhoff voltage law* (KVL) equations based on the fundamental loops associated with a chosen forest $\mathcal{F}$ are

$$\mathbf{K}_E\mathbf{V} = \mathbf{0} \qquad (3)$$

and the *Kirchhoff current law* (KCL) equations based on the fundamental cut sets associated with $\mathcal{F}$ are

$$\mathbf{Q}_E\mathbf{I} = \mathbf{0} \qquad (4)$$

We denote the waves incident into the connection circuit with $\mathbf{a}$ and the waves scattered by the connection circuit with $\mathbf{b}$. The vectors are given by

$$\mathbf{a} = \frac{1}{2}\left(\mathbf{g}^{-1}\mathbf{V} + \mathbf{g}\mathbf{I}\right) \qquad \mathbf{b} = \frac{1}{2}\left(\mathbf{g}^{-1}\mathbf{V} - \mathbf{g}\mathbf{I}\right) \qquad (5)$$

where $\mathbf{g}$ is a diagonal matrix formed by the square roots of the characteristic impedances assigned to the branches of the connection circuit.

We introduce the connection matrix $\mathbf{\Gamma}$, relating the waves $\mathbf{a}$ flowing into the connection circuit to the waves $\mathbf{b}$ scattered from the connection circuit by $\mathbf{b} = \mathbf{\Gamma}\mathbf{a}$. The matrix $\mathbf{\Gamma}$ is given by

$$\mathbf{\Gamma} = \left(\mathbf{g}\mathbf{Q}_E\mathbf{g}^{-1} + \mathbf{g}^{-1}\mathbf{K}_E\mathbf{g}\right)^{-1}\left(\mathbf{g}\mathbf{Q}_E\mathbf{g}^{-1} - \mathbf{g}^{-1}\mathbf{K}_E\mathbf{g}\right) \qquad (6)$$

3 The Circuit Scattering and Correlation Matrices

A linear noisy circuit is completely described by its signal transmission matrix and the correlation matrix of the noise sources. For stationary noise signals amplitude spectra may only be defined for the cut time signals [3]. In the following a subscript T denotes the time windowing of the signal in the interval $[-T, T]$. We subdivide the vectors $\mathbf{a}_T$ and $\mathbf{b}_T$ describing the amplitudes of the waves flowing into the connection circuit and scattered by the connection circuit into the vectors $\mathbf{a}_{1T}$ and $\mathbf{b}_{1T}$ assigned to the external ports of the connection circuit and the vectors $\mathbf{a}_{2T}$ and $\mathbf{b}_{2T}$ describing the waves flowing from and to the connection circuit. The circuit elements are described by

$$\mathbf{a}_{2T} = \mathbf{S}^{El}\mathbf{b}_{2T} + \mathbf{a}_{2T}^{El} \qquad (7)$$

where $\mathbf{S}^{El}$ is the circuit element S–matrix and the vector $\mathbf{a}_{2T}^{El}$ represents the equivalent noise wave sources of the circuit elements. The power spectra of these noise wave sources are given by the correlation matrix

$$\mathbf{C}^{El} = \lim_{T\to\infty} \frac{1}{2T}\left\langle \mathbf{a}_{2T}^{S}\mathbf{a}_{2T}^{S\dagger}\right\rangle \qquad (8)$$

By appropriate numbering of the circuit element ports $\mathbf{S}^{El}$ and $\mathbf{C}^{El}$ are of block diagonal type, where each block corresponds to a circuit element. If a linear passive circuit element described by the S–matrix $\mathbf{S}_i$ exhibits only thermal noise the corresponding correlation matrix is given by

$$\mathbf{C}_i^S = \frac{1}{2}\mathrm{kT}_0\left(\mathbf{1} - \mathbf{S}_i\mathbf{S}_i^\dagger\right) \qquad (9)$$

whereas the correlation matrices of the active circuit elements may be obtained from measured noise parameters [3]. From eq. (7) we obtain the circuit equations in tableau form

$$\begin{bmatrix} \mathbf{\Gamma}_{11} & -\mathbf{1} & \mathbf{\Gamma}_{12} & \mathbf{0} \\ \mathbf{\Gamma}_{21} & \mathbf{0} & \mathbf{\Gamma}_{22} & -\mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & -\mathbf{S}^{El} \end{bmatrix} \begin{bmatrix} \mathbf{a}_{1T} \\ \mathbf{b}_{1T} \\ \mathbf{a}_{2T} \\ \mathbf{b}_{2T} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{a}_{2T}^{El} \end{bmatrix} \qquad (10)$$

The solution of the tableau equations (10) by matrix inversion is

$$\begin{aligned} \mathbf{b}_{1T} &= \left[\mathbf{\Gamma}_{11} + \mathbf{\Gamma}_{12}\left(\mathbf{1} - \mathbf{S}^{El}\mathbf{\Gamma}_{22}\right)^{-1}\mathbf{S}^{El}\mathbf{\Gamma}_{21}\right]\mathbf{a}_{1T} \\ &\quad + \mathbf{\Gamma}_{12}\left(\mathbf{1} - \mathbf{S}^{El}\mathbf{\Gamma}_{22}\right)^{-1}\mathbf{a}_{2T}^{El} \end{aligned} \qquad (11)$$

From this we obtain the S–matrix $\mathbf{S}$ and the correlation matrix $\mathbf{C}^S$ of the multiport

$$\mathbf{S} = \mathbf{\Gamma}_{11} + \mathbf{\Gamma}_{12}\left(\mathbf{1} - \mathbf{S}^{El}\mathbf{\Gamma}_{22}\right)^{-1}\mathbf{S}^{El}\mathbf{\Gamma}_{21} \qquad (12)$$

$$\mathbf{C}^S = \mathbf{M}\mathbf{C}^{El}\mathbf{M}^\dagger \qquad (13)$$

with the transformation matrix $\mathbf{M}$ given by

$$\mathbf{M} = \mathbf{\Gamma}_{12}\left(\mathbf{1} - \mathbf{S}^{El}\mathbf{\Gamma}_{22}\right)^{-1} \qquad (14)$$

$\mathbf{M}^{\dagger}$ is the Hermitean conjugate of $\mathbf{M}$. For the numerical solution of the tableau equations advanced methods will be used instead of matrix inversion.

For the circuit elements given in Table 1 the calculated S-parameters and noise parameters are depicted in Figs. 3, 4 and 5.

4 Conclusion

Compared with known methods based upon the S-matrix representation there are no restrictions in the circuit topology, and compared with topological methods based on signal description by voltage and current amplitudes this methods yields well conditioned equations for any linear circuit elements and for an arbitrary choice of of the circuit forest.

Branch	Element
3,4	line $50 * \sqrt{2}\Omega$
5	C= 5.03pF
6	L= 4.78nH
7,8	NE64535
9	L= 10.8nH
10	C= 1.67pF
11,12	line $50 * \sqrt{2}\Omega$
13,14	line $50 * \sqrt{2}\Omega$
15	C= 5.03pF
16	L= 4.78nH
17,18	NE64535
19	L= 10.8nH
20	C= 1.67pF
21,22	line $50 * \sqrt{2}\Omega$
23	R= 100Ω
24	R= 100Ω
	$Z_0 = 50\Omega$

Table 1: Circuit elements of the amplifier circuit

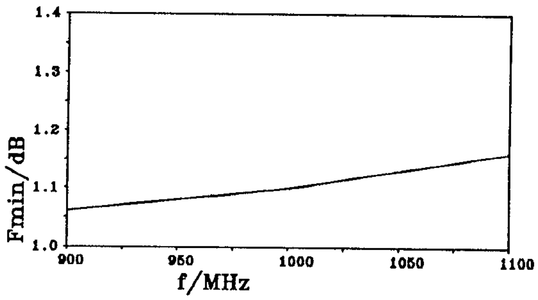

Figure 4: Minimum Noise Figure

References

[1] P. Russer, "Evaluation of the S-Matrix of Microwave Circuits with General Topology," to be published.

[2] H. Hillbrand, P. Russer, "An Efficient Method for Computer Aided Noise Analysis of Linear Amplifier Networks," *IEEE Trans. Circuits and Systems* **CAS–23**, 235–238 (1976).

[3] P. Russer, S. Müller, "Noise Analysis of Linear Microwave Circuits," *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, Nr. 3, pp. 287–316, 1990.

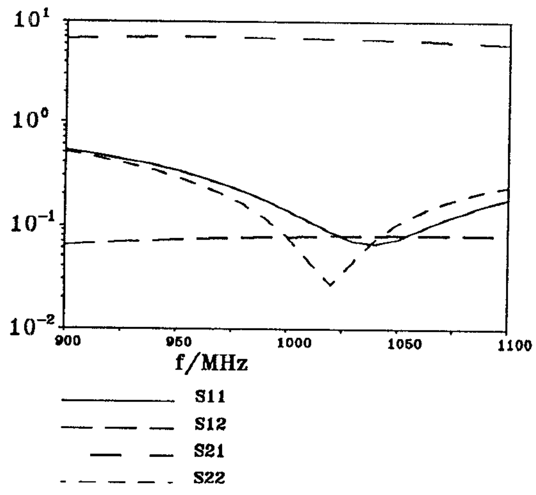

Figure 3: Magnitude of S Parameters

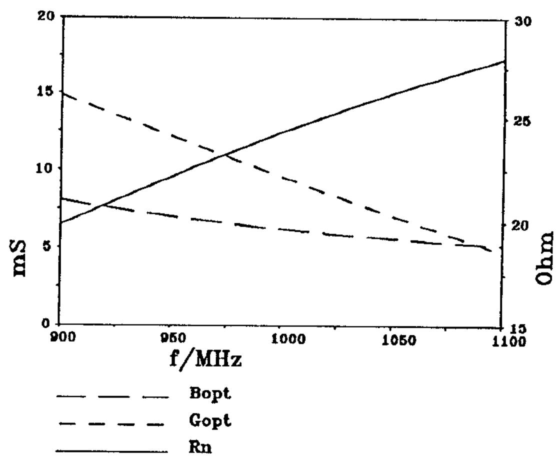

Figure 5: Parameters $Y_{opt} = B_{opt} + jG_{opt}$ and R_n

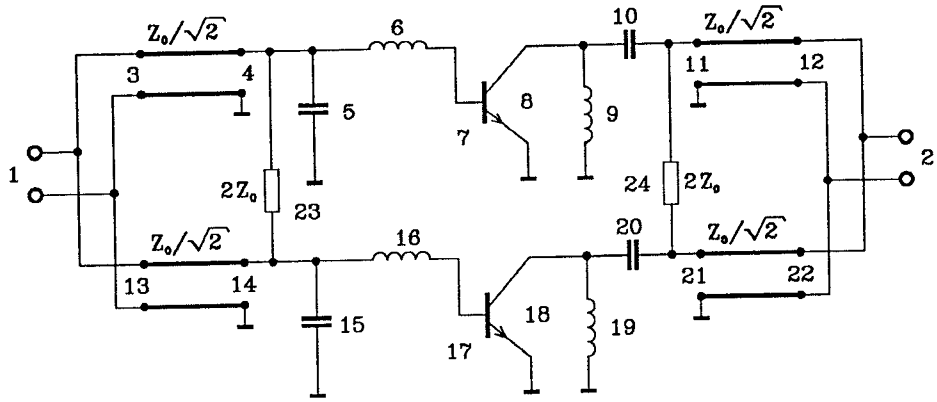

Figure 1: Amplifier circuit

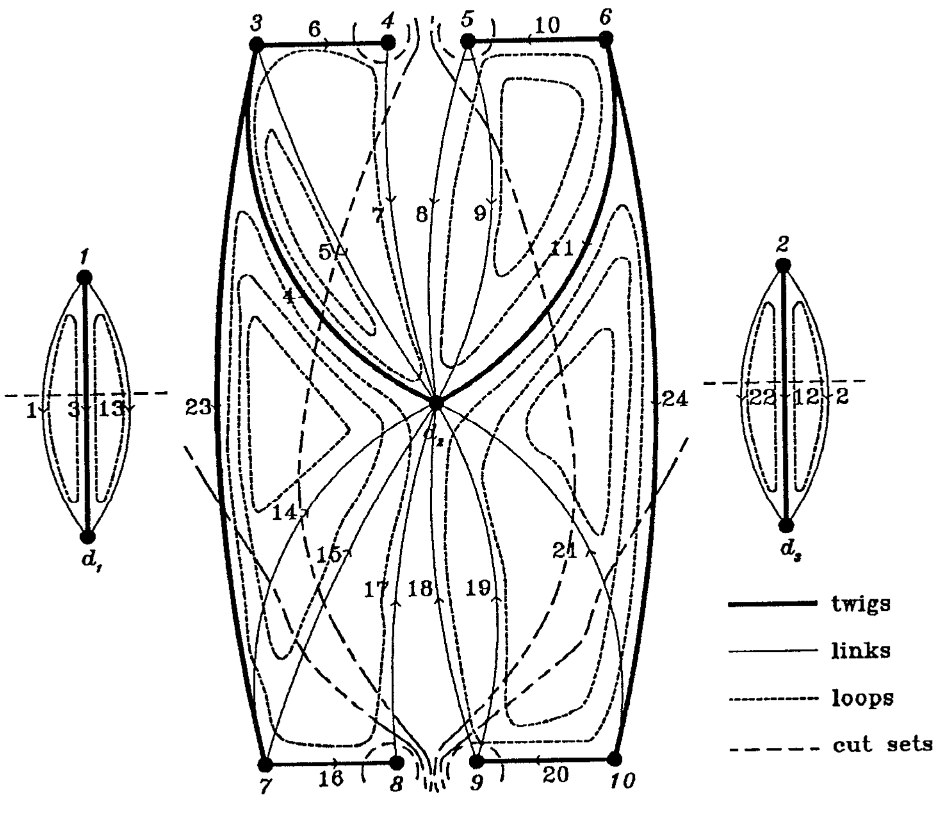

Figure 2: The circuit graph